# Stony Brook University
# The Graduate School

Doctoral Defense Announcement

## Abstract

Complete Set of Eigenfunctions of

The Quantum Periodic Toda Chain

By

**Daniel An**

The quantum periodic Toda chain is a system of $N$ particles whose quantum behavior is governed by the Hamiltonian operator

$$H = -\frac{1}{2}\sum_{k=1}^{N}\frac{\partial^2}{\partial x^2} + \sum_{k=1}^{N-1} e^{q_k - q_{k+1}} + e^{q_N - q_1}.$$

Building on the previous works of Gutzwiller(1980) and Sklyanin(1985), Pasquier and Gaudin(1992) was able to find quantization conditions for this system by introducing an integral transform which turned the Schrodinger equation into the Baxter equation. They gave the solution for the Baxter equation, but were not able to state how to obtain the actual eigenfunctions due to the lack of any inverse transform. Kharchev and Lebedev(1999) succeeded in constructing a more explicit integral transform and its inverse, which they used to prove that Pasquier-Gaudin solutions can be inverted to give an $L^2$ eigenfunction for the quantum periodic Toda chain Hamiltonian. However, they did not know whether these solutions formed a complete set.

We answer this question affirmatively, that all eigenfunctions of the quantum periodic Toda chain arise from the Pasquier-Gaudin solutions, in the form of integral representation obtained explicitly by Kharchev and Lebedev. This will, in addition, show that the joint spectrum of commuting Hamiltonians of the Periodic Toda chain is simple

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